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An Enhanced Two-Phase Fuzzy Programming Model for Multi-Objective Supplier Selection Problem

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ABSTRACT

Supplier selection is an essential task within the purchasing function of supply chain management because it provides companies with opportunities to reduce various costs and realize stable and reliable production. However, many companies find it difficult to determine which suppliers should be targeted as each of them has varying strengths and weaknesses in performance which require careful screening by the purchaser. Moreover, information required to assess suppliers is not known precisely and typically fuzzy in nature. In this paper, therefore, fuzzy multi-objective linear programming (fuzzy MOLP) is presented under fuzzy goals: cost minimization, service level maximization and purchasing risk. To solve the problem, we introduce an enhanced two-phase approach of fuzzy linear programming for the supplier selection. In formulated problem, Analytical Hierarchy Process (AHP) is used to determine the weights of criteria, and Taguchi Loss Function is employed to quantify purchasing risk. Finally, we provide a set of alternative solution which enables decision maker (DM) to select the best compromise solution based on his/her preference. Numerical experiment is provided to demonstrate our approach.

Keywords: Supplier Selection, Two-Phase Fuzzy Programming, Supply Chain

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1. INTRODUCTION

In global and competitive market, the need for establishing a longer-term relationship that fosters cooperation among suppliers and their customers has been highlighted. However, many purchasers find it difficult to determine which suppliers should be targeted as each of them has varying strengths and weaknesses in performance. Moreover, the importance of each criterion tends to vary from one purchaser to others. This problem becomes more complicated as the simultaneous evaluation is required in terms of qualitative and quantitative criteria. So, every decision needs to be integrated by trading off performances of different suppliers at each supply chain stage.

One of the main characteristic of supplier selection is that this task is characterized by an imprecision and incomplete of data which results in vagueness of information related to decision criteria. Stochastic models are usually based on representation of existing uncertainty by probability concepts and are, consequently, limited to tackling the uncertainties captured (Aliev *et al.*, 2007). Moreover, the estimation of probability distribution is difficult to carry out in a fuzzy environment because of the imprecision of the data. This is why, Fuzzy set theory (FST) is applied as an appropriate tool to handle this problem effectively.

Li *et al*. (2005) proposed a two-phase approach to compute efficient solutions of fuzzy programming as an improvement of compromise approach. In their model, they proposed that minimum acceptable achievement level of fuzzy objectives and constraints is set to the solution of max-min operator (Zimmermann approach). However, this method may not necessarily yield a feasible solution when the minimum acceptable achievement level is closer or equal to the most optimistic value (closer to 1). In this research, an enhanced two-phase fuzzy programming for multi-objective supplier selection problem has been developed as decision support tool for DMs in order to select the best compromise solution based on his/her preference regarding the assignment of order quantity to the selected supplier(s).

2. LITERATURE REVIEW

A number of studies have been devoted to examining supplier selection methods. Quantitative techniques have become increasingly applied recently. A comprehensive review of numerous quantitative techniques used for supplier selection has been done by Weber *et al*. (1991). They found that linear weighting models, mathematical programming models and statistical/probabilistic approaches have been most applied approaches.

Some researches used single objective, such as cost, to evaluate suppliers. Kaslingam and Lee (1996) developed an integer programming model to select suppliers and to determine order quantities with the objective of minimizing total supplying costs which include purchasing and transportation costs. Caudhry *at al.* (1993) used linear and mixed binary integer programming to minimize aggregate price considering both all unit and incremental quantity discount.

As an extension of single objective techniques, multi-objective mathematical programming has been proposed to solve a more complex supplier selection problem. Weber *et al.* (1998) combined multi-objective programming (MOP) and Data Envelope Analysis (DEA) to deal with non-cooperative supplier negotiation strategies where the selection of one supplier results in another being left out of the solution. Dahel (2003) studied multi-objective mixed integer programming to select supplier and allocate product to them in multi-product environment. Xia and Wu (2007) improved AHP using rough set theory and multi-objective mixed integer programming to determine the best suppliers and optimal quantity allocated to each of them in the case of multiple sourcing, multiple product with multiple criteria. Kokangul and Susuz (2009) proposed an integration of analytical hierarchy process (AHP) and non-linear integer MOP to determine the best supplier and optimal order quantity among them that simultaneously maximize total value of purchase and minimize total cost of purchase. Chamodrakaz *et al.* (2010) provided new approach of two-stage supplier selection problem. At the first stage, an initial screening is performed through the enforcement of hard constraint on the selection criteria, and in the second stage, final selection is performed using a modified variant of fuzzy preference programming (FPP). Eroll and Farell (2003) used qualitative and quantitative factor in supplier selection. A fuzzy QFD (Quality function Deployment) is used to translate linguistic input into qualitative data and then combine it with other quantitative data to develop a multi-objective mathematical programming model.

This paper focuses on fuzzy multi-objective linear programming (fuzzy MOLP) to deal with supplier selection problem. Kumar *et al.* (2006) developed a fuzzy multi-objective integer programming approach for vendor selection problem subject to constraints including buyer's demand, vendors' capacity, and derived an optimal solution using max-min operator (Zimmermann's approach). To evaluate the performance of the model, they perform sensitivity analysis on the order allocation and objective function by changing the degree of uncertainty in vendor capacity. Amid *et al.* (2006) solved fuzzy MOLP supplier selection problem by applying weighted additive method to facilitate an asymmetric fuzzy decision making technique. Since they found the performance of such a method is not adequate to support decision making process, α-cut approach is then proposed to improve the resulted achievement level. Later on, Amid *et al.* (2010) applied weighted max-min approach in supplier selection problem and compared the performance of the proposed approach with max-min operator and weighted additive model. They found that the ratio of achievement level of objectives matches the ratio of the objectives weight.

Although there were a number of publications adopting fuzzy programming model in supplier selection problem in the literature, most of them rely on the application of the existing method and very few researches have concerned with the improvement in the methodological process of deriving optimal solution. Kagnicioglu (2006) proposed super-transitive approximation to determine the weights of objectives and constraint in formulating fuzzy MOLP model in supplier selection and solved the model using max-min operator and weighted additive model. Yucel and Guneri (2010) proposed a new method of weights calculation in fuzzy MOLP supplier selection. Both researches mentioned above only focus on the process for weights calculation for fuzzy objective and constraints.

It has been approved that solving fuzzy MOLP using max-min operator may not result in a optimal solution (Tseng and Chen, 1998; Dubois and Fortemps, 1999; Lin, 2004). Such a lack has been resolved by Li *et al.* (2005) who proposed two-phase approach to compute efficient solutions of fuzzy MOLP problems as the improvement of compromise approach of Wu *et al.* (2001). *Li et al.* (2005) found that the performance of compromise approach decreases when the DM prefers to choose the minimum acceptable achievement level closer or equal to the most optimistic value. In their proposed method, minimum acceptable achievement level is set to the solution of max-min operator. In this sense, the performance of compromise approach can be improved and, on the other hand, the disadvantage of max-min operator can be overcome. However, the two-phase approach will face the same obstacle if max-min operator outputs the result closer or equal to the most optimistic value, and hence, cannot provide the improvement. To release the above-mentioned shortcomings and to help obtain a more reasonable compromise solution, therefore, this paper proposes an enhanced two-phase approach of fuzzy MOLP by introducing additional variables which control the relaxation of resulted overall achievement level and apply it to solve supplier selection problem.

In the proposed supplier selection model, net cost minimization, service level maximization and purchasing risk minimization are incorporated as fuzzy goals. The first two criteria are cited most often in ordering decision (Ghodsypour and O'Brien, 1998). Purchasing risk is included as one objective to measure the risk of potential loss incurred if purchaser allocates a certain amount of product to purchase to a certain supplier. To this end, Taguchi loss function (TLF) is used to quantify this risk. AHP is employed to determine relative important between fuzzy goals and constraints.

The rest of the paper is organized as follows. The comprehensive description of the proposed model is described in section 3. It includes the theoretic descriptions for Taguchi Loss Function, AHP, Fuzzy Multi-objective Linear Programming, the proposed "an enhanced two-phase approach" to solve fuzzy MOLP. This section is closed with the "solution prodedures" which describes step by step procedures to solve fuzzy MOLP supplier selection problem. Then a numerical experiment is presented in Section 4. Finally, the paper is concluded in section 5.

3. THE PROPOSED INTEGERATED METHOD

This section presents all methods involved in our fuzzy MOLP model. First, Taguchi loss function is described to quantify the risk associated with purchasing decision, followed by AHP to calculate a relative importance of sub-criteria used to measure risk as well as the relative importance between objectives and constraints in the final formulation. Next, fuzzy MOLP supplier selection model and an enhanced two-phase approach are presented.

3.1 Taguchi Loss Function

In traditional system, the product is accepted if the quality measurement falls within the specification limit. Otherwise, the product is rejected. The quality losses occur only when the product deviates beyond the specification limits, therefore becoming unacceptable (Pi and Low, 2005). Taguchi suggests a narrower view of quality acceptability by indicating that any deviation from quality's target value results in a loss. If the quality measurement is the same as the target value, the loss is zero. Otherwise, the loss can be measured using a quadratic function (Kathley and Waler, 2002).

There are three types of Taguchi loss functions: "target is best" (two-sided *equal* specification or two-sided *unequal* specification), "smaller is better" and "larger is better." If $L(y)$ is the loss associated with a particular value of quality y, m is the target value of the specification, and k is the loss coefficient whose value is constant depending on the cost at the specification limits and the width of the specification, then for "target is best-two sided equal specification" type, "target is best-two sided unequal specification" type, "smaller is better" type, and "larger is better" type, the formulation of $L(y)$ are given is eq.(1)-(4), respectively.

$$L(y) = k(y-m)^2 \quad (1)$$

$$L(y) = k_1(y-m)^2 \text{ or } L(y) = k_2(y-m)^2 \quad (2)$$

$$L(y) = k(y)^2 \quad (3)$$

$$L(y) = k / y^2 \quad (4)$$

3.2 Analytical Hierarchy Process

The analytic hierarchy process (AHP) was developed to provide a simple but theoretically multiple-criteria methodology for evaluating alternatives (Saaty, 1980). The major reasons for applying AHP are because it can handle both qualitative and quantitative criteria and because it can be easily understood and applied by the DMs. AHP involves the principles of decomposition, pair-wise comparisons, and priority vector generation and synthesis.

The procedures of AHP to solve a complex problem involve six essential steps (Lee, 1999): define the unstructured problem and state clearly the objectives and outcomes; decompose the problem into a hierarchical structure with decision elements (e.g., criteria and alternatives); employ pair-wise comparisons among decision elements and form comparison matrices; use the eigen value method to estimate the relative weights of the decision elements; check the consistency property of matrices to ensure that the judgments of decision makers are consistent; and aggregate the relative weights of decision elements to obtain an overall rating for the alternatives.

3.3 Fuzzy Multi-objective Linear Programming

A linear multi-objective problem can be stated as: find vector x in the transformed form $x^T = |x_1, x_2, \cdots,$

$X_n|$ which minimize objective function Z_k and maximize objective function Z_l with

$$Z_k = \sum_{i=1}^{n} c_{ki} x_i, \quad k = 1, 2, \cdots, p \tag{5}$$

$$Z_l = \sum_{i=1}^{n} c_{li} x_i, \quad l = p+1, p+2, \cdots, q \tag{6}$$

subject to:

$$x \in X_d, \; X_d = \{x \mid g(x) \le b_r, \quad r = 1, 2, \cdots, m\} \tag{7}$$

where X_d is the set of feasible solution that satisfy the set of system constraints.

Zimmermann (1978) first adopted fuzzy programming model proposed by Bellman and Zadeh (1970) into conventional LP problems. The fuzzy formulation for eq. (5)-(7) can be stated as

$$Z_k = \sum_{i=1}^{n} c_{ki} x_i \le\sim Z_k^0, \quad k = 1, 2, \cdots, p \tag{8}$$

$$Z_l = \sum_{i=1}^{n} c_{li} x_i \ge\sim Z_l^0, \quad l = p+1, p+2, \cdots, q \tag{9}$$

subject to:

$$\tilde{g}_r(x) = \sum_{i=1}^{n} a_{ri} x_i \le\sim b_r, \quad r = 1, 2, \cdots, h \tag{10}$$

$$g_p(x) = \sum_{i=1}^{n} a_{pi} x_i \le b_p, \quad p = h+1, \cdots, m \tag{11}$$

$$x_i \ge 0, \quad i = 1, 2, \cdots, n \tag{12}$$

The above fuzzy MOLP is characterized by linear membership function whose value changes between 0 and 1. The membership function (μ) for fuzzy objectives are given as

$$\mu_{Z_k}(x) = \begin{cases} 1 & \text{if} \quad Z_k(x) \le Z_k^{\min} \\ \dfrac{Z_k^{\max} - Z_k(x)}{Z_k^{\max} - Z_k^{\min}} & \text{if} \quad Z_k^{\min} \le Z_k(x) \le Z_k^{\max} \\ 0 & \text{if} \quad Z_k(x) \ge Z_k^{\max} \end{cases} \tag{13}$$

$$\mu_{Z_l}(x) = \begin{cases} 0 & \text{if} \quad Z_l(x) \le Z_l^{\min} \\ \dfrac{Z_l(x) - Z_l^{\min}}{Z_l^{\max} - Z_l^{\min}} & \text{if} \quad Z_l^{\min} \le Z_l(x) \le Z_l^{\max} \\ 1 & \text{if} \quad Z_l(x) \ge Z_l^{\max} \end{cases} \tag{14}$$

and linear membership function for fuzzy constraints is given as

$$\mu_{gr}(x) = \begin{cases} 0 & \text{if} \quad g_r(x) \ge b_r + d_r \\ \dfrac{1 - (g_r(x) - b_r)}{d_r} & \text{if} \quad b_r \le g_r(x) \le b_r + b_r, \\ 1 & \text{if} \quad g_r(x) \le b_r \end{cases} \tag{15}$$

d_r is subjectively chosen tolerance interval expressing the limit of the violation of the rth inequalities constraints. In the above formulation, $Z_k^{\max}$, $Z_l^{\max}$, $Z_k^{\min}$ and $Z_l^{\min}$ mean the maximum value (worst solution) and the minimum value (best solution) of Z_k and Z_l, respectively. They are obtained through solving a single objective optimization problem respectively under each objective function (Lai and Hwang, 1994).

Zimmermann (1978) proposed a max-min operator approach to solve the above fuzzy MOLP. The Eq. (5)~(7) can be transformed into the following crisp formulation by introducing additional variable λ which represent an overall achievement level for both fuzzy objectives and constraints.

$$\text{Max} \quad \lambda \tag{16}$$

subject to:

$$\lambda \le \mu_{zj}(x), \; j = 1, 2, \cdots, q$$
$$\lambda \le \mu_{gr}(x), \quad r = 1, 2, \cdots, h$$
$$g_p(x) \le b_p, \; p = h+1, \cdots, m$$
$$x_i \ge 0, \quad i = 1, 2, \cdots, n \quad \text{and} \quad \lambda \in [0, 1]$$

3.4 Enhanced two-Phase Fuzzy Programming

Li *et al.* (2005) proposed a two-phase approach to compute efficient solutions of fuzzy MOLP as the improvement of compromise approach of Wu *et al.* (2001). The steps of two-phase approach are as follow:

Step 1: Solve the max-min operator problem and output the optimal value, say x^0.

Step 2: Set the lower bound $\lambda_j^l = \mu_{zj}(x^0)$ for objective function and $\gamma_r^l = \mu_{gr}(x^0)$ for fuzzy constraints and solve the following model to get a final solution x.

$$\text{Max} \sum_{j=1}^{q} \omega_j \lambda_j + \sum_{r=1}^{h} \beta_r \gamma_r \tag{17}$$

subject to:

$$\lambda_j^l \le \lambda_j \le \mu_{zj}(x), \; j = 1, 2, \cdots, q$$
$$\gamma_r^l \le \gamma_r \le \mu_{gr}(x), \quad r = 1, 2, \cdots, h$$
$$g_p(x) \le b_p, \; p = h+1, \cdots, m$$
$$x_i \ge 0, \quad i = 1, 2, \cdots, n \quad \text{and} \quad \lambda_j, \gamma_r \in [0, 1]$$
$$\sum_{j=1}^{q} \omega_j + \sum_{r=1}^{h} \beta_r = 1, \quad \omega_j, \beta_r \ge 0$$

It should be noted that the value of minimum acceptable achievement level is a compromise preference value of decision maker. However, this method may not necessarily yield a feasible solution when the minimum acceptable achievement level is closer or equal to the most optimistic value. Moreover, due to the problem structure of supplier selection under consideration, formulating linear programming requires a careful parameter setting because selection criteria are quantified using wide range of numerical input. Inappropriate parameter setting may also result in infeasible solution. To release the above-mentioned shortcomings and to help obtain a more reasonable compromise solution, therefore, this research proposes an enhanced two-phase approach of fuzzy MOLP. Namely, we propose to solve the following model to get the final solution *x*:

$$\text{Max} \ \ (1-p)\left(\sum_{j=1}^{q}\omega_j\lambda_j+\sum_{r=1}^{h}\beta_r\gamma_r\right)-p\left(\sum_{j=1}^{q}\varepsilon_j+\sum_{r=1}^{h}\delta_r\right) \quad (18)$$

subject to:

$$\lambda_j^l-\varepsilon_j\le\lambda_j\le\mu_{zj}(x),\ j=1,\ 2,\cdots,\ q$$

$$\gamma_r^l-\delta_r\le\gamma_r\le\mu_{gr}(x),\ \ r=1,2,\cdots,h$$

$$g_p(x)\le b_p,\ p=h+1,\cdots,m$$

$$x_i\ge 0,\ \ i=1,2,\cdots,n \quad \text{and} \quad \lambda_j,\gamma_r\in[0,1]$$

$$\sum_{j=1}^{q}\omega_j+\sum_{r=1}^{h}\beta_r=1;\ \omega_j,\ \beta_r\ge 0;\ 0\le\varepsilon_j\le\lambda_j^l;\ \ 0\le\delta_r\le\gamma_r^l$$

where ε_j and δ_j are augmented variables to relax the overall achievement level resulted from the foregoing max-min operator problem, respectively, and *p* is a weighting factor which control the original objective function value and the relaxation value. Apparently, it is desirable such relaxation is as small as possible as long as the feasibility is hold.

3.5 Supplier Selection Problem

In this section, we formulate a mathematical model of fuzzy MOLP supplier selection. The following notations are defined in order to describe the model.

i = index for supplier ($i = 1, 2, \cdots, N$)
D = demand of buyer (unit)
B = total budget of buyer to purchase product ($)
x_i = order quantity to supplier *i* (unit)
p_i = unit price of supplier *i* ($)
f_i = service level of supplier *i* (% fulfillment)
r_i = purchasing risks of supplier *i* (% risk)
C_i = capacity of supplier *i* (unit)

The MOLP model for supplier selection is as follow:

$$\text{Min} \ \ Z_1=\sum_{i=1}^{n}p_ix_i \quad (19)$$

$$\text{Max} \ Z_2=\sum_{i=1}^{n}f_ix_i \quad (20)$$

$$\text{Min} \ \ Z_3=\sum_{i=1}^{n}r_ix_i \quad (21)$$

subject to:

$$\sum_{i=1}^{n}x_i=D \quad (22)$$

$$\sum_{i=1}^{n}p_ix_i\le B \quad (23)$$

$$x_i\le C_i \quad (24)$$

$$x_i\ge 0 \quad (25)$$

Eq. (19) minimizes the net cost for ordering product to satisfy demand. Eq. (20) maximizes the service level of suppliers. Eq. (21) minimizes the purchasing risk when the firm allocates a certain amount of product to purchase to a certain supplier. Eq. (22) puts restriction that order quantity assigned to suppliers must satisfy the total demand. Eq. (23) ensures that the total cost of purchasing does not exceed the amount of budget allocated by the firm. Eq. (24) guarantees that the order quantity assigned to each supplier will not exceed supplier capacity limit. Eq. (25) is non-negativity constraint.

3.6 Solution Procedures

The proposed fuzzy MOLP supplier selection problem is constructed through the following steps:

Step 1: Define the criteria for supplier selection problem

Step 2: Construct the MOLP supplier selection problem according to defined criteria (minimize purchasing cost, maximize service level, and minimize purchasing risk) and constraint of the buyer and suppliers. The purchasing risk is quantified as followed:

a. Define sub-criteria
b. Measure the relative important of sub-criteria using AHP
c. For each sub-criteria, define a target value, calculate loss coefficient and Taguchi loss
d. Find weighted Taguchi loss by employing the output of AHP. This value is used in MOLP model as the coefficient of objective of minimizing purchasing risk

Step 3: Find membership function for each criteria and constraint.

a. Determine a lower bound of each objective by solving MOLP as a single objective supplier selection problem using each time only one objective.
b. As in a), determine an upper bound of each objective by solving MOLP as a single objective supplier selection problem using each time only one objective.

Step 4: Calculate relative importance of criteria and constraints using AHP.

Step 5: Reformulate the MOLP supplier selection into equivalent crisp model using the enhanced two-phase fuzzy MOLP and find the set of feasible solution.

4. NUMERICAL EXAMPLE

Suppose that one firm should manage three suppliers for one product. Management wants to improve the efficiency of the purchasing process by evaluating their suppliers. The management considers three objective functions i.e. minimizing net cost, maximizing service level and minimizing purchasing risk subject to constraint regarding demand of product, supplier capacity limitation, firm's budget allocation, etc. The estimated value of suppliers' net price, service level and suppliers' capacity are given in Table 1. An allocated budget of the firm to purchase the product is $20,000. The demand is a fuzzy number and is predicted to be about 1400 unit with refraction of-100 and 150 unit.

Table 1. Suppliers' Quantitative Information.

	Net Cost/unit ($)	Service Level (% fill rate)	Capacity (unit)
Supplier 1	10	75	500
Supplier 2	12	90	600
Supplier 3	9	85	550

Purchasing risk is measured from four sub-criteria: quality, order fulfillment, on-time delivery, and distance/proximity. Concerning product quality, DM sets the target value of defect products at zero and the upper specification limit at 3% to indicate the allowable deviation from the target value. Zero loss will occur for 0% defective parts and 100% loss will occur at the specification limit of 3% defective parts. For order fulfillment rate, the loss will be zero for the supplier who fulfills all order quantity (100%) and the total loss will occur if supplier can only satisfy 80% of total order. For on-time delivery, the specification limit of delivery is 10 days and 5 days for early and delay shipment, respectively. The DM will tolerate the shipment for maximum 5 days delay and 10 early. In this case, manufacturer will incur 100% loss if shipment is 5 days delayed or 10 days earlier from scheduled shipment, and on contrary, no loss incurred if the shipment is on time. For distance/proximity, a zero-loss will occur at the closest supplier and the specification limit is up to 40% of the closest supplier. It means that the manufacturer will incur 100% loss if there is other suppliers in consideration whose distance reaches the specification limit. The specification limit and range value of each sub-criterion are presented in Table 2.

Calculating the value of k from Eq. (1)~(4) gives 1111.11, 0.64, and 6.25 for quality, order fulfillment, distance/proximity, respectively. For on-time delivery, $k_1 = 4$ and $k_2 = 1$ (since an unequal two side specification is considered for on-time delivery, there exists two loses coefficients, k_1 and k_2).

The actual values (Table 3), together with the value of loss sub-criterion k previously calculated for these four sub-criteria, are used to calculate the individual Taguchi Loss for each supplier for each criterion using Eq. (1)~(4). For example, the actual quality value of supplier A is 1.0% defective rate, which means 1.0% deviation from the target value. Individual Taguchi loss is then calculated by entering this value into Eq. (1)~(4). The result is shown in Table 4.

Suppose the pair-wise comparison matrix and local weight for each of these four sub-criteria using AHP is that shown in Table 5. The consistency Ratio (CR) of table 5 is 0.0971 (less than 0.1). The weighted Taguchi loss is then calculated using Taguchi losses and the local weight of sub-criterion. Table 4 shows the weighted Taguchi loss and the normalized Taguchi loss for each supplier. The normalized Taguchi loss is then used as a coefficient of purchasing risk in fuzzy MOLP. Based on suppliers' data in Table 1 and the normalized Taguchi Loss in Table 4, the fuzzy MOLP supplier selection of the presented problem is constructed according to Eq. (8)-(12) as follow:

Table 2. The Specification Limit and Range Value of Four Sub-criteria.

Criteria	Target Value	Range	Specification Limit
Quality(% defect rate)	0%	0~3%	3%
Order fulfillment	100%	80%~100%	80%
On-time delivery(days)	0	-10 to 0 and 0 to 5	10 days earlier, 5 days delay
Distance/Proximity(miles)	The closest	0~40%	40%

Table 3. Actual Value of the Four Sub-criteria.

Supplier	Quality (% defect)	Order Fulfillment (% unit)	On-time delivery (days)	Distance/Proximity (miles)
1	1.0%	90%	2	6
2	1.2%	95%	4	7
3	1.5%	97%	-1	9

Table 4. Taguchi Loss.

Supplier	Quality	Order Fulfillment	On-time delivery	Distance/ Proximity	Weighted Taguchi Loss	Normalized Taguchi Loss
1	11.11	79.01	16.00	0.00	34.078	0.284
2	16.00	70.91	64.00	18.06	43.627	0.363
3	25.00	68.06	1.00	156.25	42.411	0.353
				Total	120.116	1.000

Table 5. Pair-wise Comparison Matrix

Sub-criteria	Quality	Order Fulfillment	On-time delivery	Distance /Proximity	Local Weight
Quality	1	2	2	5	0.417
Order Fulfillment	1/2	1	3	5	0.334
On-time delivery	1/2	1/3	1	5	0.191
Distance/Proximity	1/5	1/5	1/5	1	0.058
Criteria	Net price	Service Level	Purchasing Risk	Demand	Local Weight
Net Cost	1	2	3	3	0.447
Service Level	1/2	1	2	3	0.282
Purchasing risk	1/3	1/2	1	2	0.164
Demand	1/3	1/3	1/2	1	0.106

$$\text{Min}\, Z_1 = 10x_1 + 12x_2 + 9x_3 \leq\sim Z_1^0$$
$$\text{Max}\, Z_2 = 0.75x_1 + 0.9x_2 + 0.85x_3 \geq\sim Z_2^0$$
$$\text{Min}\, Z_3 = 0.284x_1 + 0.363x_2 + 0.353x_3 \leq\sim Z_3^0$$
subject to:
$$x_1 + x_2 + x_3 \cong 1400$$
$$10x_1 + 12x_2 + 9x_3 \leq 20000$$
$$x_1 \leq 500,\ x_2 \leq 600,\ x_3 \leq 550$$
$$x_i \geq 0$$

The criteria and constraint can be considered equally important and added together for comparison. However, such a comparison is generally unfair due to certain criteria that that may be more important than others. In this model, the weight of cost, service level, purchasing risk and demand are derived from AHP. Table 5 shows the pair-wise comparison matrix and local weights for criteria and constraint. The consistency Ratio (CR) is 0.026 (less than 0.1).

Calculating the membership function using max-min operator (Eq. (16)) gives 0.566, 0.566 and 0.566 for $\mu_{z1}(x^0)$, $\mu_{z2}(x^0)$, and $\mu_{z3}(x^0)$, respectively. The crisp formulation of the above fuzzy MOLP using the enhanced two-phase approach according to Eq. (18) is given as

$$\text{Max}\ (1-\rho)\left(0.447\lambda_1 + 0.282\lambda_2 + 0.164\lambda_3 + 0.106\gamma_1\right) - p\left(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \delta_1\right)$$
subject to:
$$0.566 - \varepsilon_1 \leq \lambda_1 \leq \frac{14900 - \left(10x_1 + 12x_2 + 9x_3\right)}{750}$$
$$0.566 - \varepsilon_2 \leq \lambda_2 \leq \frac{14900 - \left(10x_1 + 12x_2 + 9x_3\right)}{750}$$
$$0.566 - \varepsilon_3 \leq \lambda_3 \leq \frac{\left(0.75x_1 + 0.90x_2 + 0.85x_3\right) - 1158}{37}$$
$$0.847 - \delta_1 \leq \gamma_1 \leq \frac{1550 - \left(x_1 + x_2 + x_3\right)}{150}$$
$$12x_1 + 16x_2 + 12x_3 \leq 20000$$
$$x_1 \leq 500,\ x_2 \leq 600,\ x_3 \leq 550$$
$$\omega_1 + \omega_2 + \omega_3 + \beta_1 = 1$$
$$\varepsilon_1 \leq 0.566,\ \varepsilon_2 \leq 0.566,\ \varepsilon_3 \leq 0.566,\ \delta_1 \leq 0.847$$
$$\lambda_1,\ \lambda_2,\ \lambda_3,\ \gamma_1 \in [0, 1]$$
$$x_1, x_2, x_3, \omega_1, \omega_2, \omega_3, \beta_1 \geq 0$$

In this problem, the original two-phase approach fails to yield the feasible solution. The constraint associated with λ_1 cannot be satisfied because the value of λ_1 equal to 0.526 which is lower than the designated value of its lower bound ($\lambda_1^l = 0.566$).

Table 6 provides a set of the feasible solutions resulted by utilizing the proposed method which includes the overall achievement level, individual achievement level, ordering plan and the objective value of the equivalent crisp model along with the upper and lower bounds of fuzzy objectives and constraint. As shown in the Table 6, the overall achievement level of the proposed approach is known to be better than that of Max-min operator (λ = 0.566) when value of p is lower than 0.5. When p is equal or greater than 0.5, the overall achievement level decreases. A lower p value indicates the model attempts to find a solution by relaxing more the critical objective related to the corresponding constraint to achieve a better achievement level of the other objective.

In this model, the service level (Z_2) and the purchasing risk (Z_3) are a critical objectives as the corresponding constraints are relaxed for almost any p value (critical constraint). This implies that the model tends to sacrifice the performance of these objectives because it is at less of cost decreasing the performance of these objectives rather than decreasing other. The greatest relaxation is occurred when p is 0.10. The achievement level of Z_2 is totally relaxed (μ_{z2} = 0) to achieve a better achievement level for Z_1 followed by Z_3. The achievement level of Z_2 reaches the best possible value for the entire value of p when Z_3 is relaxed for p is equal to 0.54 and 0.6. Moreover, Z_1 is free from relaxation as it is the most important objective, whose assigned weight is the highest, according to the DM's preference ($\omega_1 >> \omega_2 > \omega_3$).

In this fuzzy formulation, all suppliers are selected to supply product to the firm. Moreover, upon more careful observation, it is revealed that ordering to Supplier 1 and Supplier 3 is more preferable. It is indicated from the order quantity assigned to these suppliers as they receive the biggest amount of order quantity which is equal/closer to their full capacity. In this case, it is not profitable to order more quantity to supplier 2 because it offers the most expensive price and the highest purchasing risk among others. As mentioned above, the price (net cost) is put as the main concern of the DM (the highest weight). Thus, placing a smaller order quantity to Supplier 2 is the best decision.

Without loss of generality, suppose that the DM wants to select p equals 0.10. In this solution, μ_{z1} and μ_{z3} improve to 0.991 and 0.980, respectively, which

Table 6. Comparison of Max-min Operator and the Proposed Approach.

	Max-min Operator	**The Proposed Method**						
		p = 0.10	p = 0.30	p = 0.54	p = 0.60	p = 0.70	p = 0.90	p = 1.00
Objective Function	-	0.608	0.529	0.456	0.439	0.413	0.360	0.334
First Term of obj. function *(Overall achievement level)*	λ = 0.566	0.665	0.545	0.467	0.444	0.419	0.368	0.342
Second Term of obj. function *(Overall relaxation degree)*	-	0.570	0.022	0.021	0.009	0.008	0.008	0.008
μ_{z1}	0.566	0.991	0.630	0.616	0.570	0.570	0.570	0.570
μ_{z2}	0.566	0.000	0.548	0.570	0.570	0.566	0.566	0.566
μ_{z3}	0.566	0.980	0.570	0.549	0.561	0.566	0.566	0.566
ε_1	-	0.000	0.000	0.000	0.000	0.000	0.000	0.000
ε_2	-	0.570	0.022	0.000	0.000	0.004	0.004	0.004
ε_3	-	0.000	0.000	0.021	0.900	0.004	0.004	0.004
x_1	500	500	499	499	499	500	500	500
x_2	390	351	374	379	389	389	389	389
x_3	533	550	550	550	550	535	534	534
Z_1	14475.42	14156.67	14,427.15	14,437.87	14,472.50	14,472.50	14,472.50	14,472.50
Z_2	1178.95	1158.00	1178.29	1179.09	1179.09	1178.94	1178.94	1178.94
Z_3	471.60	463.40	471.60	472.02	471.78	471.69	471.69	471.69
Membership Function	$\mu = 0$	$\mu = 1$	$\mu = 0$					
Net Cost (Z_1)	-	14150	14900					
Service Level (Z_2)	1158	1195	-					
Purchasing Risk (Z_3)	-	463	483					
Demand	1300	1400	1550					

results in the best value of Z_1 and Z_3. However, the DMs should carefully notice that the achievement level of service level, the second most important criteria, declines toward the worst performance (μ_{z2} = 0). Eventually, final decision should be made by the DM to choose the most favorable decision among the feasible alternative solutions according to his/her preference.

5. CONCLUSION

Supplier selection is an essential task within the purchasing function that needs careful screening under some qualitative and quantitative criteria. Moreover, most information required to assess supplier is usually not known precisely and typically fuzzy in nature over the planning horizon. Concerning such characteristics, this research proposes integrated methodology for FMOLP model for supplier selection. In formulated problem, the most common fuzzy objectives and parameter in practical ordering decision have been presented. AHP is used to avoid the subjective judgment on qualitative/quantitative criteria and TLF is employed to quantify the purchasing risk. For the purpose of solving the FMOLP problem, the enhanced two-phase fuzzy programming model has been developed. Through numerical experiment, we demonstrate the promising advantage of our proposed approach over the max-min operator (Zimmermann's approach). Finally, this integrated approach provides a set of potential feasible solutions which guide DMs to select the best solution according to their preference. This also refers to a multi-objective optimization problem that should be concerned in future studies.

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APPENDIX

The membership functions for objective functions and demand constraint.

$$\mu_{Z_1}(x) = \begin{cases} 1 & \text{if} \quad Z_1 \le 14150 \\ \dfrac{14900 - Z_1}{750} & \text{if} \quad 14150 \le Z_1 \le 14900 \\ 0 & \text{if} \quad Z_1 \ge 14900 \end{cases} \tag{26}$$

$$\mu_{Z_2}(x) = \begin{cases} 0 & \text{if} \quad Z_2 \le 1158 \\ \dfrac{Z_2 - 1158}{1195 - 1158} & \text{if} \quad 1158 \le Z_2 \le 1195 \\ 1 & \text{if} \quad Z_2 \ge 1195 \end{cases} \tag{27}$$

$$\mu_{Z_3}(x) = \begin{cases} 1 & \text{if} \quad Z_3 \le 463 \\ \dfrac{483 - Z_3}{483 - 463} & \text{if} \quad 463 \le Z_3 \le 483 \\ 0 & \text{if} \quad Z_3 \ge 483 \end{cases} \tag{28}$$

$$\mu_{gd}(x) = \begin{cases} \dfrac{d(x) - 1300}{100} & \text{if} \quad 1300 < d(x) < 1400 \\ \dfrac{1550 - d(x)}{150} & \text{if} \quad 1400 < d(x) < 1550 \\ 0 & \text{if} \quad d(x) \le 1300 \text{ and } d(x) \ge 1550 \end{cases} \tag{29}$$

where $d(x) = x_1 + x_2 + x_3$